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Fluid flow through a channel bounded by parallel flat walls

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In this paper, we have considered the unsteady laminar flow of a viscous incompressible fluid through a channel bounded by two parallel flat walls. In section 1, the basic equations are given and the velocity distribution for the oscillatory flow calculated, which has been utilized to find the expression for the flux flow, drag, work-expended, energy-utlized and the efficiency. In section 2, temperature field has been discussed for the oscillatory motion when (i) the walls are kept at a constant temperature (ii) the wall temperature varies as an oscillatory function of time.

Formulation of the problem

The equations of conservation of mass, momentum and energy for two dimensional flow are (Lal, in press)

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta x} = 0 \qquad \text{...(1.1)}$$

$$\rho\left(\frac{\delta u}{\delta t}+u\,\frac{\delta u}{\delta x}+v\,\frac{\delta u}{\delta y}\right) = -\,\frac{\delta p}{\delta x}+\mu\left(\frac{\delta^2 u}{\delta x^2}+\frac{\delta^2 u}{\delta y^2}\right) \qquad \text{...(1.2)}$$

$$\rho\,\left(\frac{\delta v}{\delta t}+u\,\frac{\delta v}{\delta x}+v\,\frac{\delta u}{\delta y}\right) = -\frac{\delta p}{\delta y}+\mu\,\left(\frac{\delta^2 v}{\delta x^2}+\frac{\delta^2 v}{\delta y^2}\right) \qquad \text{...(1.3)}$$

$$g\rho c\,\left(\frac{\delta T}{\delta t}+u\,\frac{\delta T}{\delta x}+v\frac{\delta T}{\delta y}\right) = k\left(\frac{\delta^2 T}{\delta x^2}+\frac{\partial^2 T}{\partial y^2}\right)+2\mu\phi \qquad \text{...(1.4)}$$

where

$$\phi = \left(\frac{\delta u}{\delta x}\right)^2+\left(\frac{\delta v}{\delta y}\right)^2+\frac{1}{2}\left(\frac{\delta v}{\delta x}+\frac{\delta u}{\delta y}\right)^2 \qquad \text{...(1.5)}$$

Let the flow be along the x-axis so that $v = 0$ and consequently $\frac{\delta u}{\delta x}=0$ from equation (1.1). The equation (1.2) together with equation (1.1) reduces to

$$\frac{\delta u}{\delta t} = -\frac{1}{\rho}\,\frac{\delta p}{\delta x}+\nu\,\frac{\delta^2 u}{\delta y^2} \qquad \text{...(1.6)}$$

The equation (1.6) has been solved elsewhere (Lal, in press) by taking the following forms for the pressure gradient and consequently velocity distribution. He has calculated the velocity distribution which is given below for further use in the paper.

Let

$$\left.\begin{aligned} &-\frac{1}{\rho}\,\frac{\delta p}{\delta x}=Ke^{\iota\omega t} \\ \text{and}\qquad &u(y,t)=f(y)e^{\iota\omega t} \end{aligned}\right\} \qquad \text{...(1.7)}$$

where real part of both equations are to be considered and making substitutions from equation (1.7) into (1.6), we have after some simplifications

$$\frac{d^2f}{dy^2}+A^2f=B \qquad .. (1.8)$$

where

$$A^2=-\frac{\iota\omega}{\nu} \text{ and } B=-\frac{K}{\nu} \qquad .. (1.9)$$

Solution of equation (1.8) is

$$f=c_1 \sin Ay + c_2 \cos Ay + \frac{B}{A^2} \qquad .. (1.10)$$

where c_1 and c_2 are arbitrary constants of integration. Now, using the boundary conditions as

$$u = 0 \text{ at } y = \pm y_0 \text{ (on the walls for all values of } t\text{)} \qquad . (1.11)$$

and consequently

$$f = 0 \text{ at } y = \pm y_0 \qquad .. (1.12)$$

Equation (1.10) gives

$$\left.\begin{aligned} f &= \frac{B}{A^2}\left(1-\frac{\cos Ay}{\cos Ay_0}\right) \\ \text{and} \qquad u(y,t) &= \frac{B}{A^2}\left(1-\frac{\cos Ay}{\cos Ay_0}\right)e^{\iota\omega t} \end{aligned}\right\} \qquad .. (1.13)$$

where the real part of the equation gives the required expression for u. The discharge of flux per second as given earlier (1) is

$$Q=R\int_{-y_0}^{y_0} u\,dy=Re^{\iota\omega t}\int_{-y_0}^{y_0} \frac{B}{A^2}\left(1-\frac{\cos Ay}{\cos Ay_0}\right)dy \qquad ...(1.14)$$

The above expression has not been separated earlier into the real and imaginary parts. We have made such calculations and the result is given below. From equation (1.14), we get

$$Q = \frac{2K}{\omega}y_0 \sin \omega t + \frac{(2\nu)^{1/2}K}{\omega^{3/2}} \frac{\{\sinh(2\sqrt{\omega/2\nu}y_0) - \sin(2\sqrt{\omega/2\nu}y_0)\}\cos \omega t}{\cos(2\sqrt{\omega/2\nu}y_0) + \cosh(2\sqrt{\omega/2\nu}y_0)}$$

$$-\frac{(2\nu)^{1/2}K}{\omega^{3/2}} \cdot \frac{\{\sin(2\sqrt{\omega/2\nu}y_0) + \sin(2\sqrt{\omega/2\nu}y_0)\}\sin \omega t}{\cos(2\sqrt{\omega/2\nu}y_0) + \cosh(2\sqrt{\omega/2\nu}y_0)}$$

and for small ω, we have ...(1.15)

$$Q_1 = \frac{2Ky_0^3}{3\nu} - \frac{K\omega^2t^2y_0^3}{3\nu} \qquad ...(1.16)$$

Certain calculations for discharge of flux, drag, work-expended, energy-utilized and efficiency have been deduced for the oscillatory flow which has not been discussed earlier by Lal (In Press).

For the boundary conditions

$$\left.\begin{array}{l} u = U = f_1 e^{i\omega t} (f = f_1) \text{ at } y = y_0 \\ f = 0 \text{ at } y = -y_0 \end{array}\right\} \qquad \ldots(1.17)$$

and

for all time $t > 0$ we get

$$f = \frac{f_1}{2}\left(\frac{\sin Ay}{\sin Ay_0} + \frac{\cos Ay}{\cos Ay_0}\right) + \frac{B}{A^2} \qquad \ldots(1.18)$$

and thus considering the real part in (1 18) the discharge of flux becomes

$$Q_2 = \frac{f_1\sqrt{\nu}}{\sqrt{2\omega}} \frac{\{\sinh 2\sqrt{\omega/2\nu}y_0) + \sin (2\sqrt{\omega/2\nu}y_0)\}\cos \omega t}{\cos (2\sqrt{\omega/2\nu}y_0) + \cosh (2\sqrt{\omega/2\nu}y_0)}$$

$$-\frac{f_1\sqrt{\nu}}{\sqrt{2\omega}} \frac{\{\sin(2\sqrt{\omega/2\nu}y_0) - \sinh(2\sqrt{\omega/2\nu}y_0)\} \sin \omega t}{\cos (2\sqrt{\omega/2\nu}y_0) + \cosh (2\sqrt{\omega/2\nu}y_0}$$

$$+\frac{2Ky_0 \sin \omega t}{\omega} + \frac{(2\nu)^{1/2}K}{\omega^{3/2}} \frac{\{\sinh(2\sqrt{\omega/2\nu}y_0) - \sin (2\sqrt{\omega/2\nu}y_0)\}\cos \omega t}{\cos (2\sqrt{\omega/2\nu}y_0) + \cosh (2\sqrt{\omega/2\nu}y_0)}$$

$$-\frac{(2\nu)^{1/2}K}{\omega^{3/2}} \cdot \frac{\{\sin (2\sqrt{\omega/2\nu}y_0) + \sinh (2\sqrt{\omega/2\nu}y_0)\}\sin \omega t}{\cos (2\sqrt{\omega/2\nu}y_0) + \cosh (2\sqrt{\omega/2\nu}y_0)} \qquad \ldots(1.19)$$

For small ω, we have

$$Q_3 = f_1 y_0 + \frac{2}{3\nu} K y^3{}_0 + \omega^2\left(-\frac{f_1 t^2 y_0}{2} + \frac{f_1 y^3{}_0 t}{3\nu} - \frac{K t^2 y^3{}_0}{3\nu}\right) + \omega^4\left(\frac{-f_1 t^3 y^3{}_0}{18\nu}\right) \qquad (1.20)$$

The traction per unit width and along the length l, of the upper wall (Dryden *et al* 1932) is

$$l\mu\left(\frac{\partial u}{\partial y}\right)_{y=y_0} = \frac{l\mu f_1}{2} A(\cot Ay_0 - \tan Ay_0) e^{i\omega t} + \frac{\mu l B}{A} \tan Ay_0 e^{i\omega t} \qquad \ldots(1.21)$$

Considering real part of the above equation, we get for drag

$$D = -\frac{\mu l f_1\sqrt{\omega}}{2\sqrt{2\nu}} \frac{\{\sin (4\sqrt{\omega/2\nu}y_0) + \sinh (4\sqrt{\omega/2\nu}y_0)\} \cos \omega t}{\cos^2(2\sqrt{\omega/2\nu}y_0) - \cosh^2(2\sqrt{\omega/2\nu}y_0}$$

$$-\frac{\mu l f_1\sqrt{\omega}}{2\sqrt{2\nu}} \frac{\{\sin (4\sqrt{\omega/2\nu}y_0) - \sin(4\sqrt{\omega/2\nu}y_0\}) \sin \omega t}{\cos^2 (2\sqrt{\omega/2\nu}y_0) - \cosh^2(2\sqrt{\omega/2\nu}y_0)}$$

$$-\frac{K\mu l}{\sqrt{2\nu\omega}} \cdot \frac{(\cos \omega t - \sin \omega t). \sin(\sqrt{\omega/2\nu}y_0}{(\cos 2\sqrt{\omega/2\nu}y_0) + \cosh(2\sqrt{\omega/2\nu}y_0)}$$

$$-\frac{K\mu l}{\sqrt{2\nu\omega}} \frac{(\cos \omega t + \sin \omega t). \sinh(2\sqrt{\omega/2\nu}y_0)}{\cos (2\sqrt{\omega/2\nu}y_0) + \cosh (2\sqrt{\omega/2\nu}y_0)} \qquad \ldots(1.22)$$

The work-expended is

$$U_1 = R\left\{\frac{f_1\mu l A U}{2}(\cos Ay_0 - \tan Ay_0)e^{i\omega t} + \frac{\mu l B U}{2}\tan Ay_0\, e^{i\omega t}\right\}$$

$$= -\frac{\mu l f_1^2\sqrt{\omega}}{2\sqrt{2\nu}}\frac{\{\sin(4\sqrt{\omega/2\nu}y_0) + \sinh(4\sqrt{\omega/2\nu}y_0)\}\cos 2\omega t}{\cos^2(2\sqrt{\omega/2\nu}y_0) - \cosh^2(2\sqrt{\omega/2\nu}y_0)}$$

$$-\frac{\mu l f_1^2\sqrt{\omega}}{2\sqrt{2\nu}}\frac{\{\sin(4\sqrt{\omega/2\nu}y_0) - \sinh(4\sqrt{\omega/2\nu}y_0)\}\sin 2\omega t}{\cos^2(2\sqrt{\omega/2\nu}y_0) - \cosh^2(2\sqrt{\omega/2\nu}y_0)}$$

$$-\frac{Kf_1\mu l}{\sqrt{2\nu\omega}}\frac{(\cos 2\omega t - \sin 2\omega t).\sin(2\sqrt{\omega/2\nu}y_0)}{\cos(2\sqrt{\omega/2\nu}y_0) + \cosh(2\sqrt{\omega/2\nu}y_0)}$$

$$-\frac{Kf_1\mu l}{\sqrt{2\nu\omega}}\frac{(\cos 2\omega t + \sin 2\omega t)\sinh(2\sqrt{\omega/2\nu}y_0)}{\cos(2\sqrt{\omega/2\nu}y_0) + \cosh(2\sqrt{\omega/2\nu}y_0)} \qquad \text{.. (1.22)}$$

While the energy-utilized is

$$E = \mu l Q_2\left(-\frac{1}{\rho}\frac{\partial p}{\partial x}\right) = \mu l Q_2(Ke^{i\epsilon t}) \qquad \text{.. (1.23)}$$

Considering the real part of the equation (1.24) we get for E_1 as

$$E_1 = \frac{K\mu l f_1\sqrt{\nu}}{\sqrt{2\omega}}\frac{\{\sin(2\sqrt{\omega/2\nu}y_0) + \sinh(2\sqrt{\omega/2\nu}y_0)\}\cos 2\omega t}{\cos(2\sqrt{\omega/2\nu}y_0) + \cosh(2\sqrt{\omega/2\nu}y_0)}$$

$$-\frac{K\mu l f_1\sqrt{\nu}}{\sqrt{2\omega}}\frac{\{\sin(2\sqrt{\omega/2\nu}y_0) - \sinh(2\sqrt{\omega/2\nu}y_0)\}\sin 2\omega t}{\cos(2\sqrt{\omega/2\nu}y_0) + \cosh(2\sqrt{\omega/2\nu}y_0)}$$

$$+\frac{2K^2\mu l y_0}{\omega}\sin 2\omega t$$

$$+\frac{(2\nu)^{1/2}K^2\mu l}{\omega^{3/2}}\frac{\{\sinh(2\sqrt{\omega/2\nu}y_0) - \sin(2\sqrt{\omega/2\nu}y_0)\}\cos 2\omega t}{\cos(2\sqrt{\omega/2\nu}y_0) + \cosh(2\sqrt{\omega/2\nu}y_0)}$$

$$-\frac{(2\nu)^{1/2}K^2\mu l}{\omega^{3/2}}\frac{\{\sin(2\sqrt{\omega/2\nu}y_0) + \sinh(2\sqrt{\omega/2\nu}y_0)\}\sin 2\omega t}{\cos(2\sqrt{\omega/2\nu}y_0) + \cosh(2\sqrt{\omega/2\nu}y_0)} \qquad \text{.. (1.25)}$$

The efficiency η is given by

$$\eta = \frac{E_1}{U_1} \qquad \text{...(1.26)}$$

where E_1 and U_1 are as in equations (1.23) and (1.25)

Temperature Distribution

Lal (In Press) has studied the temperature field for exponential flow. Oscillatory flow has more important application in engineering problems and hence we have considered the energy equation when the wall temperature varies as an oscillatory function of time. The basic energy equation (3) is

$$\frac{\partial T}{\partial t} = k' \frac{\partial^2 T}{\partial y^2} + k'' \left(\frac{\partial u}{\partial y}\right)^2 \qquad \text{...(2.1)}$$

where

$$k' = \frac{k}{g\rho c} \text{ and } k'' = \frac{\mu}{g\rho c} \qquad \text{.. (2.2)}$$

Substituting for $\frac{\partial u}{\partial y}$ from equation (1.13) into (2.1), we have

$$\frac{\partial T}{\partial t} = k' \frac{\partial^2 T}{\partial y^2} + k'' \frac{B^2}{A^2}\left(\frac{\sin^2 Ay}{\cos^2 Ay_0}\right) e^{2\iota\omega t} \qquad \text{...(2.3)}$$

To solve the equation (2.3), we take

$$T = f(y) e^{2\iota\omega t} \qquad \text{...(2.4)}$$

Substituting from equation (2.4) into (2.3), we have

$$\frac{d^2 f}{dy^2} + A_1^2 f = A_2 \sin^2 Ay \qquad \text{...(2.5)}$$

where

$$A_1^2 = -\frac{2\iota\omega}{k'} \text{ and } A_2 = \frac{k''}{k'} \times \frac{-B^2}{A^2} \cdot \frac{1}{\cos^2 Ay_0} = \text{const.} \qquad \text{...(2.6)}$$

The solution of equation (2.5) is

$$f = c_1 \cos A_1 y + c_2 \sin A_1 y - \frac{A_2}{2A_1^2} - \frac{A_2}{2}\frac{\cos 2Ay}{A_1^2 - 4A^2} \qquad \text{...(2.7)}$$

where c_1 and c_2 are constants of integration. Assuming the boundary conditions as

$$\left.\begin{aligned} T &= T_1 \text{ at } y = 0 \\ T &= T_0 \text{ at } y = \pm y_0 \end{aligned}\right\} \text{ when } t \text{ is finite, say, } t > 0 \qquad \text{...(2 8)}$$

and consequently

$$\left.\begin{aligned} T_1 &= f_1 e^{2\iota\omega t} (f = f_1) \text{ at } y = 0 \\ T_0 &= f_0 e^{2\iota\omega t} (f = f_0) \text{ at } y = \pm y_0 \end{aligned}\right\} \text{ when } t \text{ is finite, say, } t > 0 \qquad \text{...(2.9)}$$

($f_0 = 0$, if the walls are kept at constant temperature for all time $t > 0$)
From the first boundary condition, we get

$$c_2 = 0,\ c = f_1 + \frac{2 A_2 A^2}{A_1^2 (A_1^2 - 4A^2)} \qquad \text{...(2.10)}$$

Thus, we get

$$f = \left\{f_1 + \frac{2 A_2 A^2}{A_1^2 (A_1^2 - 4A^2)}\right\} \cos A_1 y + \frac{A_2}{2}\left\{\frac{1}{A_1^2} - \frac{\cos 2Ay}{A_1^2 - 4A^2}\right\} \qquad \text{...(2.11)}$$

So that

$$T = \left\{ f_1 + \frac{2A_2 A^2}{A_1^2 (A_1^2 - 4A^2)} \right\} \cos A_1 y e^{2_i \omega t} + \frac{A_2}{2} \left\{ \frac{1}{A_1^2} - \frac{\cos 2Ay}{A_1^2 - 4A^2} \right\} e^{2_i \omega t} \quad \text{...(2.12)}$$

From the second boundary conditions, we have

$$\left.\begin{aligned} f_0 &= c_1 \cos A_1 y_0 - c_2 \sin A_1 y_0 + \frac{A_2}{2A_1^2} - \frac{A_2}{2} \frac{\cos 2Ay_0}{A_1^2 - 4A^2} \\ f_0 &= c_1 \cos A_1 y_0 + c_2 \sin A_1 y_0 + \frac{A_2}{2A_1^2} - \frac{A_2}{2} \frac{\cos 2Ay_0}{A_1^2 - 4A^2} \end{aligned}\right\} \quad \text{...(2.13)}$$

We find that

$$c_2 = 0 \text{ and } c_1 = \left\{ f_0 - \frac{A_2}{2} \left(\frac{1}{A_1^2} - \frac{\cos 2Ay_0}{A_1^2 - 4A^2} \right) \right\} \cdot \frac{1}{\cos A_1 y_0} \quad \text{.. (2.14)}$$

and thus we get

$$T = \left\{ f_0 + \frac{A_2}{2} \cdot \frac{\cos 2Ay_0}{A_1 - 4A^2} - \frac{A_2}{2A_1^2} \right\} \frac{\cos A_1 y}{\cos A_1 y_0} e^{2_i \omega t} + \frac{A_2}{2} \left\{ \frac{1}{A_1^2} - \frac{\cos 2Ay}{A_1^2 - 4A^2} \right\} e^{2_i \omega t} \quad \text{.. (2.15)}$$

Temperature field T_1, when the walls are at a constant temperature is

$$T_1 = \left\{ \frac{A_2}{2} \cdot \frac{\cos 2Ay_0}{A_1^2 - 4A^2} - \frac{A^2}{2A_1^2} \right\} \frac{\cos A_1 y}{\cos A_1 y_0} e^{2_i \omega t} + \frac{A_2}{2} \left\{ \frac{1}{A_1^2} - \frac{\cos 2Ay}{A_1^2 - 4A^2} \right\} e^{2_i \omega} \quad \text{.. (2.16)}$$

The equation (2. 16) can be put as

$$T_1 = \frac{\{pP - qQ + MN(P_1 - P_2)\cos 2\omega t + MN(Q_1 - Q_2)\sin 2\omega t\}}{MN} + i \frac{\{pQ - qP + MN(Q_1 - Q_2)\cos 2\omega t + MN(P_1 - P_2)\sin 2\omega t\}}{MN} = P_r + iP_i \quad \text{...(2.17)}$$

where P_r stands for the real part in equation (2.17) and P_i the imaginary part.

Thus, we get the real part as

$$P_r = \frac{pP - qQ + MN(P_1 - P_2)\cos 2\omega t + MN(Q_1 - Q_2)\sin 2\omega t}{MN} \quad \text{...(2.18)}$$

where

$$P = \frac{K^2\mu k' \left\{ \begin{array}{r} \nu \cosh 2\phi \cos 2\phi\, (\sinh^2\phi \sin^2\phi - \cosh^2\phi \cos^2\phi) \\ + \sinh^2\phi \sin^2 2\phi \sinh\phi \cosh\phi \end{array} \right\} + \{(2k'-\nu)(\sinh^2\phi \sin^2\phi - \cos^2\phi \cos^2\phi)\}}{4k\omega^2\nu\,(2k'-\nu)(\cosh^2\phi\cos^2\phi + \sinh^2\phi \sin^2\phi)^2} \quad .\ (2.19)$$

$$Q = \frac{K^2\mu k' \{\nu \sinh 2\phi \sin 2\phi\, (\sinh^2\phi \sin^2\phi - \cosh^2\phi \cos^2\phi) + \cosh\phi \sinh\phi \cosh 2\phi \cos 2\phi \sin 2\phi\} + \{(2k'-\nu)(\sinh\phi \cosh\phi \cos\phi \sin\phi)\}}{4k\omega^2\nu(2k'-\nu)(\cosh^2\phi\cos^2\phi + \sinh^2\phi\sin^2\phi)^2} \quad \ldots(2.20)$$

$$p = X\cos 2\omega t - Y \sin 2\omega t \quad \ldots(2.21)$$

$$q = X \sin 2\omega t + Y\cos 2\omega t \quad \ldots(2.22)$$

$$P_1 = \frac{K^2\mu k'}{4k\omega^2\nu}\left[\frac{\cosh^2\phi\cos^2\phi - \sinh^2\phi\sin^2\phi}{(\cosh^2\phi\cos^2\phi + \sinh^2\phi\sin^2\phi)^2}\right] \quad \ldots(2.23)$$

$$P_2 = \frac{K^2\mu k'\,[\cosh 2\phi_1 \cos 2\phi_1 (\cosh^2\phi\cos^2\phi - \sinh^2\phi\sin^2\phi) + \sinh 2\phi_1 \sin 2\phi_1 \sin 2\phi \sinh\phi\cosh\phi]}{4k\omega^2(\nu - 2k')(\cosh^2\phi\cos^2\phi + \sinh^2\phi\sin^2\phi)^2}. \quad \ldots(2.24)$$

$$Q_1 = -\frac{K^2\mu k'}{2k\omega^2\nu}\,\frac{\cosh\phi\cos\phi\sin\phi\sinh\phi}{(\cosh^2\phi\cos^2\phi + \sin^2\phi\sinh^2\phi)^2} \quad (2.25)$$

$$Q_2 = \frac{K^2\mu k'\{\sinh 2\phi_1 \sin 2\phi_1 (\cosh^2\phi\cos^2\phi - \sinh^2\phi\sin^2\phi) - 2\cosh 2\phi_1\cos 2\phi_1 \cosh\phi\sinh\phi\cos\phi\sin\phi\}}{4k\omega^2(\nu - 2k')(\cosh^2\phi\cos^2\phi + \sinh^2\phi\sin^2\phi)^2} \quad \ldots(2.26)$$

$$M = \cosh^2\alpha\cos^2\alpha + \sin^2\alpha\sinh^2\alpha \quad \ldots(2.27)$$

$$N = (\cosh^2\phi\cos^2\phi + \sin^2\phi\sinh^2)\phi^2 \quad \ldots(2.28)$$

$$X = \cos\Psi\cosh\Psi\cos\alpha\cosh\alpha + \sin\Psi\sinh\Psi\sin\alpha\sinh\alpha \quad \ldots(2.29)$$

$$Y = \sin\Psi\sinh\Psi\cos\alpha\cosh\alpha - \cos\Psi\cosh\Psi\sin\alpha\sinh\alpha \quad \ldots(2.30)$$

and

$$\phi = \sqrt{\omega/2\nu}\,y_0,\ \phi_1 = \sqrt{\omega/2\nu}\,y,\ \Psi = \sqrt{\omega/k'}\,y,\ \alpha = \sqrt{\omega/k'}\,y_0 \quad \ldots(2.31)$$

To calculate the phase angle, we put equation (2.16) such as

$$T = \left\{\frac{PX - QY + MN(P_1 - P_2)}{MN} + i\,\frac{PY + QX + MN(Q_1 - Q_2)}{MN}\right\}e^{2i\omega t}$$

Considering real part in (2.32), we may write it as ...(2.32)

$$T_0 = |Z|\cos(2\omega t + \beta) \quad \ldots(2.33)$$

where

$$Z = s_r + \iota s_i$$
$$= \frac{1}{MN}\left[PX - QY + MN(P_1 - P_2) + \iota\{PY + QX + MN(Q_1 - Q_2)\}\right] \qquad \ldots(2.34)$$

and

$$\beta = \tan^{-1}\frac{s_i}{s_r} \text{ is the phase angle}$$

The wall temperature leads or lags behind the fluid temperature by a phase angle and the phase angle is zero if $s_i = 0$ and it is $\frac{\pi}{2}$ if $s_r = 0$.

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References

Dryden H. L., Murnaghan F. P. & Bateme H. 1932 *Hydrodynamics*, Dover Publication, 184.

Lal K. *Science Research Journal, B.H.U. In Press.*

Pai S. I. 1956 *Viscous flow theory, Laminar flow* (*INO*, Vol. I. 43-52.)